Секция «Теория упругости»

Identification the Order of Fractional Derivative with Respect to Caputo Operator in the Bagley-Torvik Model

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This paper is proposed for solving a partial differential equation of second order with a fractional derivative, in which the order of the fractional derivative is in the range from one to two and is not known in advance. We propose the solution of the equation is based on the separation of variables (the Fourier method). This model is utilized to characterize oscillatory processes in a viscous medium. A technique for identifying the fractional production order is given

$$a\frac{\partial^2 U(x,t)}{\partial t^2} = \frac{\partial^2 U(x,t)}{\partial x^2} + cD_{0t}^{\alpha}U(x,t) + bD_{0x}^{\beta}U(x,t), \tag{1}$$

$$U(0,t) = U(L,t) = 0 \tag{2}$$

and with initial conditions

$$U(x,0) = \varphi(x) \tag{3}$$
$$U_t(x,0) = \psi(x)$$

Here, $1 < \alpha < 2; a, c$ and b-an arbitrary constants, D^{α} is the Caputo fractional derivative of order α. To study the problem, we consider the auxiliary Cauchy problem:

$$my''(x) + bD^{\alpha}y(x) + ky(x) = 0 \tag{4}$$

$$y(0) = y'(0) = 1, \tag{5}$$

Where; b is the viscosity modulus of resin, k is the rigidity modulus of resin, α is the viscoelasticity parameter of the medium, $m = 1$ is the mass of the granule and $y(x)$ is the displacement. The main problem is the identification the order of fractional derivative, to choose the interval where the graphs of the corresponding solutions obviously do not intersect, we present the graphs of the solution of the corresponding problem for various values of the order of fractional differentiation. It follows from the figure that the interval [0.3; 0.8] can be taken for the desired interval. We take any point from this interval (in particular, you can take a point 0.5) and calculate the value obtained experimentally by a function that simulates the vibrations of polymer concrete. Obviously, from our last figure it follows that the order of the fractional operator is in the interval (1.1; 1.6), which is in good agreement with our experimental data.

References

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Illustrations

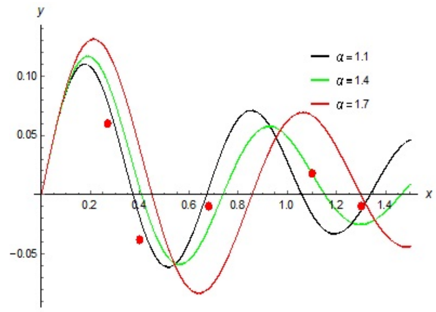

Рис. 1. Graphs of solutions to the corresponding problem for various values of the order of fractional differentiation