Normal Motion of surfaces in R^3

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Gaussian and Mean curvature flow is an example of a geometric flow of hypersurfaces in a Riemannian manifold (for example, smooth surfaces in 3-dimensional Euclidean space). A family of surfaces evolves under mean curvature flow if the normal velocity of which a point on the surface moves with its mean curvature of this surface. The mean curvature flow is a well-known geometric evolution equation. Geometric evolution problems for curves, surfaces and especially curvature flow problems are classical mathematical research fields. They lead to interesting systems of nonlinear partial differential equations and allow the appropriate mathematical modeling of physical processes such as material interface propagation, fluid free boundary motion, crystal growth. The motion of surfaces is essential and exciting objects and studied by many authors. Nakayama and Wadati [1], studied the motion of surfaces in 3-dimensional Euclidean space using the differential geometry. R. Moser [2], considered the level set formulation of the inverse mean curvature flow, and he established a connection to the problem of p-harmonic functions and gave a new proof for the existence of weak solutions. In this work, we drive the time evolution equations that are satisfied by the intrinsic quantities of the surface in R^3. Consider a surface Σ move in R^3 with position vector$X(u_1, u_2, t)$, where (u_1, u_2) are the local coordinate of the surface. The velocity of evolution is expressed as

$$\frac{dX}{dt} = \psi \; N \tag{1}$$

Where ψ is the normal velocity of the position vector of surface Σ. With study the evolution of the intrinsic quantities of the surface in R^3 (metric $g_{\mu\nu}$ and second fundamental quantities $L_{\mu\nu}$), we have

$$\frac{\partial g_{\mu\nu}}{\partial t} = -2\psi L_{\mu\nu} \tag{2}$$

$$\frac{\partial L_{\mu\nu}}{\partial t} = \nabla_\mu \nabla_\nu \psi - \psi L^k_\nu L_{\mu k} \tag{3}$$

Equations 2 and 3 give the complete description of the normal motion of the surface. Here, we will give some explicit examples about the normal motion for some surfaces as Tours and Pseudosphere, also we explain what happened for the shape of those surfaces when the normal motion depended on the Gaussian curvature or Mean curvature of these surfaces.

References

1) K. Nakayama and M. Wadati, the Motion of surfaces in the Plane, J. Phys. Soc. Jpn. 62, pp. 1895-1901, 1993.
2) R. Moser,The inverse mean curvature flow and p-harmonic functions,J. Eur. Math. Soc. 9, 77–83,2007